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Bringing big ideas in math to small children: math circles and other enrichment activities for elementary school students and teachers

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Abstract
In this talk we give a brief overview of innate mathematical knowledge and ability for children entering formal schooling and discuss the role of curriculum in continuing the development of mathematical thinking in children. We discuss the distinction between doing mathematical tasks and doing mathematics and the use of mathematics enrichment to support students in learning and doing mathematics. We then describe math circles and discuss how these can serve to introduce deep mathematics to elementary-school children and their teachers.

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Introduction

Through infancy and into the pre-school years, children develop a knowledge of, and curiosity about, mathematics. Formal school mathematics curricula may be focused on teaching a set of skills or covering specific mathematical content, but not on developing mathematical thinking and problem-solving on a deep level; furthermore, these curricula may not build on the abilities and interests that children bring to primary education. Recent curriculum reform movements, especially in the United States with the development of the Common Core State Standards, may attempt to address this, but teachers may not have the background and training to fully support this development in their students.

Enrichment activities provide an alternative to relying solely on curricula to deepen existing curiosity about and interest in mathematics. Math circles provide an example of a specific type of enrichment that give all students – and participating teachers – a deep experience of sophisticated mathematics. Math circles are informal groups of students, grouped loosely by age and interest, led by mathematicians. These groups discuss mathematical questions, often framed in open-ended ways, such as "are there numbers between numbers?" and "is there a largest number?" Students refine the questions, explore answers, argue with each other about approaches and gain tremendous insights into elegant mathematics. Math Teachers' Circles, a version of math circles specifically structured for middle and high school teachers, provide a similar experience of creative and enjoyable mathematics as a new form of professional development.

In this paper we outline some of the current concerns and issues with mathematics teaching and learning, then explore the role of math circles and Math Teachers' Circles in building excitement about and interest in mathematics to students at all ages and their teachers.

The mathematical ability of young children and the relationship to school curricula

Children, even very young children, appear to have a natural interest in and ability for mathematics. Some have argued that this is, in fact, an innate ability. The developmental psychology Karen Wynn explored the numerical cognition abilities of infants as young as 5 months old; her findings, published in 1992, supported the assertion that infants can do simple arithmetic (Wynn, 1992). In a second study that involved 9-month old infants, the researchers examined the infants' abilities to be successful with large-number operations and found that they were able to add and subtract larger numbers (McCrink and Wynn, 2004).

A study of over 1400 children in Australia, comparing mathematical knowledge at the beginning and end of the school year found that children bring considerable mathematical competence to school. As the authors note, "An important finding was that much of what has traditionally formed the mathematics curriculum for the first year of school was already understood clearly by many children on arrival at school." (Clarke, Clarke and Cheeseman, 2006).

The United States National Mathematics Advisory Panel noted the continued development of mathematical ability in children through early childhood, asserting that "Most children acquire considerable knowledge of numbers and other aspects of mathematics before they enter kindergarten." (National Mathematics Advisory Panel, 2008).

While children enter elementary school with this "considerable knowledge", it's clear that their approach to and interest in mathematics changes dramatically over time. Results from the 2011 Trends in International Mathematics and Science Study show that as students progress through school, attitudes toward mathematics changes. The results are not particularly strong at the 4th grade level, where internationally only one-third of 4th grade students said that they were "confident" about their mathematical abilities. The results for 8th graders raised further concerns: only 14% of 8th grade students, on average, said that they were confident about their mathematical ability. Furthermore, there was a large achievement gap between the percentage of "confident" 8th grade students and the much larger "not confident" group. (TIMMS, 2011)

Results from PISA 2012's international study of 15-year olds supported this connection between confidence and achievement. The study showed a strong positive correlation between openness to problem-solving and success in mathematics, as well as a negative correlation between disposition toward mathematics, and mathematics anxiety, with success in mathematics. (PISA, 2012)

Developing mathematical and problem-solving abilities in children

In 2010, the United States President's Council of Advisors on Science and Technology recommended that math and science education focus on preparing students – so that they have a strong foundation – and inspiring students – so that they are motivated to study these topics (PCAST report, 2010). The report made an important observation:

> ...studies suggest that achieving expertise is less a matter of innate talent than of having the opportunity and motivation to dedicate oneself to the study of a subject in a productive, intellectual way – and for sufficient time – to enable the brain development needed to think like a scientist, mathematician, or engineer.

The newly developed Common Core State Standards in the United States outline not only what students should know in mathematics but how students should "practice" mathematics. The standards call for a deeper development of mathematical concepts, using research- and evidenced-based approaches that align knowledge and skills with college and career expectations. The standards exemplified a shift in thinking about mathematics curriculum, moving away from a utilitarian focus on mathematics for its own sake and an emphasis on arithmetic skills. As Cuoco, Goldenberg and Mark put it, referring to mathematics curriculum:

> There is another way to think about it, and it involves turning the priorities around. Much more important than specific mathematical results are the habits of mind used by the people who create these results....The goal is not to train large numbers of high school students to be university mathematicians, but rather to allow ... students to become comfortable with ill-posed and fuzzy problems... (Cuoco, Goldenberg, Mark, 1996)

The new standards reflect this emphasis, acknowledging the importance of the development of mathematical habits of mind such as adaptive reasoning, conceptual understanding, procedural fluency, and productive disposition.

Mathematics enrichment

In addition to using mathematics curricula that reflect new knowledge about the foundational competence in mathematics that children bring to formal schooling, teachers can offer enrichment activities to better motivate children to explore mathematics and to think mathematically. Enrichment can take multiple forms and is not restricted to the school day or location:

- students receive supplemental or remedial instruction during the school day;
- students receive for advanced or accelerated instruction during the school day;
- teachers direct students to interactive mathematics websites, usually to review basic facts (for example, www.ixl.com) or to play math games (www.coolmath-games.com, for example);
- teachers offer after-school help, usually to students who need additional instruction;
- teachers organize and coach teams to participate in mathematics competitions;
- teachers encourage talented students to participate in mathematics summer programs or camps;
- schools or school districts offer "family math nights" or other community-oriented events to promote mathematics.

Many of these activities are targeted toward students with specific needs (remediation) or abilities (math competitions). While this differentiation serves a purpose, especially for students who need this support, it will not necessarily serve to build mathematical thinking and problem solving skills among all students. Indeed, enrichment, as Piggott has noted, is "almost exclusively used in the context of provision for the mathematically most able." (Piggott, 2004).

Mathematics enrichment and problem solving

Many enrichment activities are closely tied to the curriculum and are structured to support students in learning the curricular material. While students engaged in all of the activities listed above are "solving problems", they are not necessarily experiencing mathematics as a living subject, as an exploratory and dynamic discipline. And as Schoenfeld notes, problem solving is often reduced to "…sets of mathematics tasks…as vehicles of instruction, as means of practice, and as yardsticks for the acquisition of mathematical skills." He continues, "`Problem solving' has a minimal interpretation: working the tasks that have been set before you." (Schoenfeld, 1992). Mathematicians take a markedly different approach, viewing problem solving as "art" (Stanic and Kilpatrick, 1989). Halmos argues that, "the mathematician's main reason for existence is to solve problems…" (Halmos, 1980)

Math Circles

Math circles provide a new form of enrichment to students and to teachers. Historically, math circles emerged in Russia in the 1930s. Russia had a long tradition of "circles", or unofficial gatherings, that gave people an opportunity to discuss topics of common interest (Vogeli and Karp, 2010). Mathematics circles were a special type of circle, often led by university professors or graduate students. The participants were high school students, some of whom became leading research mathematicians. In some cases, the circles prepared students for Mathematical Olympiads, which were very difficult competitions for secondary students. The concept of a mathematical circle changed as it moved to the United States with Russian émigrés. Math circles, as they are called in the U.S., take many different forms. Some focus on preparing students for high-level mathematics competitions such as the International Mathematical Olympiad; others offer advanced mathematics topics to bright students, usually at the high school level; and some provide informal opportunities for interested students to use games and hands-on activities to explore mathematics (Vandervelde, 2009).

The National Association of Math Circles is a national organization in the United States that provides support and information for mathematicians and teachers interested in organizing and running math circles. Their website www.mathcircles.org lists math circle locations and websites and provides resources for problems and discussions for math circle organizers.

Math circles in the United States are often organized by a university-level mathematician, with the support of undergraduate or graduate students. Many are held at a local university, although some science museums sponsor math circles. Some circles focus on preparation for mathematics competitions, others are much more open-ended. Circles are targeted to different age groupings, with a majority of the circles designed for middle and high school students.

Math Circles for Elementary School Students

Very few math circles in the United States are offered to elementary students. There are several reasons for this. One is content: deep mathematics is sophisticated and it can be challenging to bring a five-year old to an understanding of advanced mathematical topics. Another is the classroom dynamic: teaching abstract mathematics to young children takes a great deal of patience, a fair amount of humility, and a high tolerance of chaos. But the rewards are worth it – for the students and for the instructor. As noted above, young children bring significant mathematical knowledge. They also bring a natural curiosity and a creative, open approach to learning. A math circle for elementary students brings these pieces together, giving students a deep experience of mathematics.

An example of a math circle for elementary students

From 2010 to 2013, I co-organized and ran an math circle for elementary school students in a K – 5 school in my city. The circle was open to any child in grades one through five who was interested in exploring ideas in mathematics; in particular, students did not need to be good at mathematics to join the group – all we asked for was an openness to exploring ideas and some interest in math. The objective of the program was to give students an opportunity to explore mathematical concepts and to solve interesting and unusual problems in a friendly and supportive environment.

The circle met once a week after the school day was over, for about an hour. Parents who enrolled their children paid a small fee, which covered the snacks that we provided to the children and a small stipend for the teachers who worked with us. We offered two age groupings: one class for children in the younger grades, usually first through third, and another for students in the higher grades.

Each circle had the same format: since students had just ended the school day, we gave them some time to have a snack and talk to their friends, then we started with an opening question to get them thinking about the mathematical topic. We worked on that question for about 45 minutes, then took five minutes to bring the lesson to a close and talk about the next week's lesson. The mathematical topics were chosen with several factors in mind. The topics had to be interesting and accessible and had to connect to a deeper area of mathematics. We did not try to use topics that connected to the math that students were studying in class (and in fact since we had mixed grades in the circle, this would have been very difficult). Several of the elementary school teachers partnered with us and provided useful context around mathematical knowledge that the children had as well as some support with maintaining order in the group.

A Math Circle Activity: The Handshake Problem

In Spring 2011 we spent several weeks talking with children in first, second and third grade about the Handshake Problem. The following is an overview of these lessons and the interactions.

We began by students this question: if everyone in the room shakes hands, how many handshakes in total would there be? This open-ended question led to a fierce amount of discussion, with several students shouting out what they thought the answer should be. We wrote some answers down and then asked the students to help us understand what a handshake is. For example, if two students shake hands, do we count that handshake once or twice? Is it possible to shake hands with yourself? It took some time to define the problem and narrow down the possibilities. We then asked the children the question again: if everyone in the room shook hands, how many handshakes in total would there be? One child started counting and we took that as a good sign. We asked everyone to stop and listen to what this child had to say. She told us she had counted the number of people in the room (24 people) and doubled that to get the answer. This was an excellent, if incorrect, first approach.

One of the things we told the children again and again is that one way to solve a difficult problem is to look at a small example. We demonstrated this with the handshake problem: We asked two children to come to the front of the room and shake hands, then asked the group how many handshakes they had seen. Then another child came and all three of them shook hands; again, we asked the group to count the number of handshakes. We demonstrated this for four, then five, then six children. While the children saw the answer in each case, it wasn't clear to them what the pattern was. This happens again and again in a math circle: children take steps to an answer, but may work for a long period of time with no clear answer in sight. This is an ambiguity and an uncertainty that they rarely experience within the classroom setting.

The next step was to break the children into smaller groups and ask each group to determine the number of handshakes that would result if everyone in their group shook hands. It was interesting to see the approaches they took: some very systematically tried to count handshakes, while others kept trying to find a way to multiply to get to the answer.

At this point there was a lot of discussion and a lot of energy. The children had moved from making a wild guess to thinking somewhat carefully about the problem and how to solve it. They were, for the most part, working together. We encouraged them to write down what they were discovering – some did, but not necessarily in a way that was helpful.

We pulled the group together as a large group again and asked them to tell us how many handshakes there would be if nine people shook hands. The children took a systematic approach, choosing one person and watching that person shake hands with the other eight people. That person stepped aside and there were eight people remaining. The children chose one person and watched that person shake hands with the remaining seven people. Total handshakes so far: 8 + 7. They very much wanted to add the numbers but we asked them to wait, with the idea that they may see a pattern here. Back to the handshakes: again they chose one person, who shook hands with the remaining six people. The sum became 8 + 7 + 6. Some of the students started to see what was happening now and we asked them to write down their answer but to not say it out loud. We kept going. There were six people standing and one of those people shook hands with the remaining five people. Total handshakes now: 8 + 7 + 6 + 5. We continued until only one person was standing. Most of the children saw it now: In a group of nine people, there would be 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36 handshakes.

The next step was to build on what they already knew. We asked them: if there are 10 people, how many handshakes would there be in total? Some wanted to start over, but we asked them to slow down and think about the problem differently. They already knew the answer for nine people. Now imagine that a 10th person joins the group – how many additional handshakes would there be? This was easy for them: there would be nine additional handshakes. What if an 11th person joins the group? They told us that there would be ten additional handshakes. How many handshakes in total? They saw the pattern now: 11 + 10 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 55 handshakes.

At this point we took a huge jump and asked them how many handshakes there would be if there were 100 people in the room. They immediately told us that it would be 99 + 98 + 97 + … 4 + 3 + 2 + 1 handshakes. Great! They had seen the pattern and figured out how to continue it to a number that they thought was very large (for "homework" we asked them to figure this out for 1,000 people – they were both stunned and amazed that we thought they could figure this out).

The lesson took a different turn at this point. We had moved from the very broad question about handshakes to an addition question. The children knew basic addition and some of the sat down and very carefully began adding the numbers up. We agreed this could work but we also agreed it would take a lot of time. Was there another approach? We gave them a hint by first going back to an earlier example. With ten people, we had agreed that there would be 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 . We asked them: is there a different way to add these numbers, other than using the order you see? Several of them saw right away that they could re-group the numbers (this was something they had worked in their math classes). The sum became

9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = (9 + 1) + (8 + 2) + (7 + 3) + (6 + 4) + 5

= 4(10)+5

= 55.

We took that idea back to the larger sum and let them get to work on it. This was a challenge – not only did students have to identify the groupings, but they had to move to multiplication. The first step was to help them see that

99 + 98 + 97 + … 4 + 3 + 2 + 1

could be written as

(99 + 1) + (98 + 2) + (97 + 3) + … = 100 + 100 + 100 +…

Then we had to figure out how many copies of 100 there would be. And then we had to figure out if anything was left over. This took a lot of discussion and a few visits back to the sum from 1 to 9. Eventually most of them saw that

(99 + 1) + (98 + 2) + (97 + 3)+ … = 49(100) +50.

Not all of the students knew how to multiply, so we asked those who did to explain what 49 times 100 means and how to calculate it. We finally got to the answer: 4950 handshakes. They were amazed at such a large number (much larger than any of them had initially guessed) and how we had found it. We set the homework problem for them to determine the number of handshakes for 1000 people and had a great time the next week talking about the answer and how they found it.

In the next set of lessons we re-framed the question abstractly: if there are n people in the room, how many handshakes will there be? The children knew at this point that the answer should look like

$(n-1) + (n-2) + (n-3) + \ldots + 3 + 2 + 1.$

They used the examples we had done and some geometric visualization to see that the total number of handshakes would be $\frac{1}{2}n(n-1)$.

This particular topic was a very rich one. We later used other ways to approach the same idea, using graph theory. The students enjoyed the hands-on context with the handshake question and were amazed that they could make the jump to counting handshakes for 1000 people. In a space of a few weeks, they had traveled as a group from some simple counting to a fairly abstract algebraic calculation.

Math circles and elementary school teachers

The math circle that I ran intentionally involved elementary school teachers. They were helpful in terms of organizing and keeping the children involved in the activities, but their presence was important for another reason. Their involvement in the math circle gave them an opportunity to examine one-ended questions in mathematics, experiment with approaches to solutions, and think mathematically. To prepare the for the work with the children, I met with them weekly to discuss the topic and some of the mathematics behind it.

In the United States, elementary school teachers have a very limited exposure to mathematics. Their formal education in mathematics at the post-secondary level focuses on the basic principles and concepts of school mathematics, usually in the areas of numbers/operations and geometry/measurement and, to a lesser degree, functions/algebra and statistics/probability. Mathematics education courses attempt to address the fundamental principles underlying these topics, but beyond that, elementary school teachers may not have sustained professional development with significant mathematics content. (Conference Board of the Mathematical Sciences, 2012). Furthermore, teachers' beliefs about mathematics and mathematical ability shape their teaching. Teachers who feel that all mathematics problems can be solved, or that doing mathematics is a matter of applying a set of rules to a well-defined problem, or that problem solving is equivalent to completing a set of tasks or that most people are not good at mathematics will communicate these beliefs to their students (Conference Board of the Mathematical Sciences, 2012).

A special type of math circle has recently emerged, called a Math Teachers' Circle. This type of circle is designed for practicing K – 20 teachers, primarily middle-school teachers. The model is similar to the math circles for students, in that teachers work with mathematicians on mathematical questions that are deep and sophisticated. The difference is that Math Teachers' Circles are typically structured as intensive summer workshops, usually a week long, that provide an immersive experience for the teachers who participate. During the academic year, the teachers meet monthly to continue the mathematical conversations.

There are two professional organizations in the United States dedicated to building these mathematical communities. The Special Interest Group on Math Circles for Students and Teachers is a program of the Mathematical Association of America. This group supports the vertical integration of K – 20 teachers with university faculty with the common interest of mathematics. Since the group is directly connected to one of the major mathematics professional societies in the United States, it uses national and regional meetings to showcase student math circles and math teachers' circles.

The Math Teachers' Circle Network, sponsored by the American Institute of Mathematics, has the goal of establishing 300 Math Teachers' Circles (called MTCs) by 2019. "Two primary goals of the MTC model are to encourage teachers to develop as mathematicians by engaging them in the process of doing mathematics and to create an ongoing professional K – 20 mathematics community." (Donaldson, Nakamaye, Umland and White, 2014). There are several research projects studying the benefits of Math Teachers' Circles, with encouraging initial results. In one study, results indicated that for teachers in three different Math Teachers' Circles intensive summer workshops, scores on different mathematical content areas increased significantly over the course of the workshops. The mathematical content was not directly addressed in the circle. Rather, the researchers note, "…our preliminary hypothesis…is not that teachers were gaining additional content knowledge in these areas, but rather that the experience of participating in the MTC workshop enabled them to reason more deeply about content knowledge that they already possessed." (White, Donaldson, Hodge, Ruff, 2013).

Conclusion

Math circles and Math Teachers' Circles are relatively new approaches to mathematics enrichment. Math circles can be structured so that they are accessible for all students, not simply students who excel in mathematics. In fact, the more open-ended and exploratory nature of math circles may appeal to students who don't do well in math classes in school. Through participation in math circles, children begin to understand that mathematics is a dynamic and creative enterprise. They realize that some problems do not have simple or easy answers and that they can make significant contributions, often working with others, to solve problems that at first seemed incomprehensible.

Teachers at all levels can benefit from these activities, either through working with math circle leaders or through participation in Math Teachers' Circles. This provides very different professional development for these teachers and potentially increases their mathematical confidence, their understanding of mathematics, and their openness to innovation and exploration in the classroom. Even children who don't participate in math circles can benefit if their teachers participate in Math Teachers' Circles.

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