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Research article

The effect of using a research-based curriculum on learning basic rational number concepts by Lebanese students

Iman Chahine[1,*], Thomas Post[2], Robert del Mas[2]

[1]Georgia State University, P.O. Box 3980, Atlanta, GA 30302-3980, USA
[2]College of Education & Human Development, University of Minnesota, Minneapolis, MN 55455, USA

*Corresponding author.
Email: ichahine@gsu.edu

ABSTRACT

This research study investigates the effects of using a research-based curriculum, the Rational Number Project (RNP) Curriculum, on the acquisition of basic rational number concepts by 5th grade students in contrast with those using the traditional Lebanese curriculum. The study sample comprised five schools randomly selected from a population of 14 schools. Eighteen 5th grade classrooms were randomly assigned to experimental (RNP) and control (TRAD) groups. Two instruments were used for data collection: a RNP post-test and a school test. Both tests were administered to assess students' achievement after curricular intervention. After a series of six-hour professional training sessions, RNP teachers were given a culturally relevant Arabic version of the RNP curriculum lesson plans and materials for implementation. Three techniques were employed for data analysis: reliability, fidelity and multi analysis of variance (MANOVA). Results showed that students instructed using the research-based curriculum outperformed their counterparts, who were taught fraction concepts using the Lebanese traditional curriculum, on both RNP and school tests.

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INTRODUCTION

Achievement in Lebanese schools has been decreasing steadily in the last three decades (Mahdawi, 2008). Thirty years of sporadic war and civil conflict had a devastating effect on the Lebanese education system. In the aftermath of the civil war, Lebanon witnessed a substantial out-migration of the skilled and educated that drained the human reserve base. This severely damaged schools' infrastructure and facilities in addition to weakening the overall quality of public and private educational services (World Bank, 2000).

Historically, Lebanese people have always considered education of their children as an existential necessity for survival. Since the end of the civil war in 1990, there has been a gradual recovery of the Lebanese educational system, including reconstruction and rehabilitation of school facilities, which progressed rapidly. Despite this recovery, the Lebanese school system is still saddled with the self-defeating pessimism that has accompanied decades of sectarian and ideological conflicts.

In 1998, an attempt towards building a new curriculum was put into action by the National Center for Educational Research and Development (NCERD) supported by funds from the World Bank (World Bank, 2000). The Lebanese Government launched a comprehensive educational reform including the restructuring of the education system, the design of a new K-12 curriculum, the development of new textbooks, and countrywide professional development for teachers (World Bank, 2000).

Overall, Lebanon has shown some academic advancement in mathematics and science in the past decade, particularly when compared with other countries in the Middle East. Lebanon was one of the 59 countries worldwide participating in the Trends in International Mathematics and Science Study (TIMSS) in 2007 (Mullis et al., 2008). While TIMSS assesses student learning in mathematics and science at the 4th and 8th grade levels, Lebanon participated only in the 8th grade assessment. The findings of the TIMSS 2007 report showed that students in Lebanon performed significantly below the international average in mathematics and science (the international average score is 500 for mathematics and Lebanon's score is 449) (Mullis et al., 2008). Lebanese students performed lower than the international average in all five domains tested. When compared to the 12 other participating Arab countries and authorities (Bahrain, Egypt, Iran, Jordan, Qatar, Morocco, Saudi Arabia, Syria, Tunisia, Palestine, Oman, and Yemen), Lebanon performed higher in mathematics with boys exhibiting higher achievement than girls (Mullis et al., 2008). Nonetheless, very few students taking the examination in Lebanon reached the 'advanced' or 'high international' achievement benchmark in either mathematics or science. As to attitudes towards mathematics, 63% of 8th grade Lebanese students taking the TIMSS tests indicated a positive view of mathematics and 77% valued mathematics. However, 49% had high confidence in their ability to learn mathematics, with boys indicating higher confidence than girls (Mullis et al., 2008). On average, students reported spending 10% of their mathematics lessons working on problems without teacher's guidance, while 75% of the students also reported that test questions almost always involved direct application of procedures. Lebanon is one of the 12 countries and authorities participating in TIMSS 2007, where more than half of the students attending schools come from disadvantaged backgrounds. The other countries were Algeria, Colombia, Egypt, El Salvador, Ghana, Indonesia, Morocco, the Palestinian Authority, Thailand, Tunisia, and Turkey.

Regrettably, mathematics appears to have become an obstacle to many students in Lebanon preventing them from pursuing higher educational levels. In 2007, the success rate in the Lebanese official exams did not exceed 60% (Al-Mostaqbal, 2007). This decline could be due, in part, to the narrow description of what counts as mathematics and how it is manifested in the curriculum. Calls for reform set forth by the World Bank (2000), to enhance the teaching and learning of mathematics by using research-based curricula that capitalize on problem solving and sense-making prompted the following question: What would be the impact of implementing a well-designed, research-based curriculum, accompanied by teachers' training and adequate instructional resources, on students' learning of fraction related concepts?

THEORETICAL BACKGROUND

A number of issues need to be considered when discussing the effects of implementing a research-based curriculum on the performance of 5th grade students in suburban Lebanese schools. The Lebanese traditional approach to instruction bestows the teachers with authoritative power to impart the correct rules and procedures to the students. It closely mimics the '*teaching by imposition*' instruction where concepts are acquired by strict memorization (Baroody et al., 2004). In such a

climate, the conceptual understanding of many essential mathematical concepts is sacrificed in favor of mastery of strictly mechanical manipulation of routines and algorithms.

Language represents another obstacle preventing most Lebanese students from engaging in an open dialogue that facilitates construction of meaningful knowledge. In some elementary school systems, the language of instruction is English or French whereas students' native language is Arabic. This code-shifting (changing from one language to another) unequivocally limits students' discourse and affects the quality of mathematical communication. In other schools, where the language of instruction is Arabic, another problem prevails; the language used in formal written texts is different from that used in oral communication and the Lebanese dialect that students use at home.

In particular, the teaching and learning of fraction-related concepts has been predictably challenging for Lebanese teachers and students. This is partly due to the 'multi-faceted' conception of fractions as consisting of several interconnected sub constructs (Kieren, 1980). The complexity of the content, and the dominantly teacher-centered method of instruction, inhibited student engagement and hindered meaningful understanding of various rational number concepts.

A close content analysis of a number of locally-adopted mathematics textbooks at the elementary level reveals assemblages of content extracted from foreign curricula. As a result, the succession of the topics presented does not follow logical instructional sequences or age-appropriate 'learning trajectories' that conform to children's developmental progression (Clements et al., 2004).

Ernest (2006) echoes the findings of a large body of research on the necessity of engaging learners in meaning-making and understanding and warns against the barren and ineffective transmission of semiotic systems as de-contextualized assortment of signs and rules (p. 69). Clearly, prizing the mastery of syntactic general rules inhibits students' conceptual understanding of the underlying mathematical structure and limits their access to meaningful learning.

In short, the reality of the traditional Lebanese classroom, as reflected through the locally adopted curriculum, constrains the applicability of *communicative rationality* where students and teachers are encouraged to engage in dialogical conversation that ultimately leads to construction of mathematical knowledge (Shotter, 1993). In a context where the role of teacher is unquestionably paramount, classroom discourse is skewed towards the teacher transmitting knowledge while the students receive and reiterate nuances of retained information. An unfortunate consequence of such dynamics is the attribution of students' failure to acquire, and eventually master, basic concepts to their inability to assimilate and conceptualize what is directly taught by the teachers.

The aim of this paper is to demonstrate that purposeful, well-designed, responsive and responsible instructional efforts, particularly in high poverty schools, can have positive effects on students' learning of vital mathematical concepts.

THE CURRICULA

Rational Number Project curriculum

As mentioned in Chahine (in press), the materials selected for curricular intervention included 15 learning modules developed in Arabic and based on the Rational Number Project Level 1 curriculum (Rational Number Project, 2009). The 15-lesson curriculum Rational Number Project (RNP) was specifically selected to match the content taught by the traditional curriculum and targets the use of the part-whole sub-construct as a perspective of rational numbers. Designed on the 'constructivist' model of learning, the structure of the RNP curriculum begins with content familiar to students and then builds up to new, less familiar, content.

The RNP 1 curriculum adopted in this study conforms to the Curriculum Research Framework (CRF) proposed by Clements and Sarama (2007) and adheres to its requirements. A basic tenet underlying CRF is the conformity of the research-based curricula to learning trajectories assumed to underlie children's learning and thinking.

Lebanese traditional curriculum

A systematic content analysis of the rational number topics covered in the traditional grade 5 curriculum (TRAD) reveals an imbalanced and fragmented subject content. This is in dissonance with the learning trajectories detailed in the literature on the acquisition of fractional knowledge (Steffe and Olive, 2010; Petit et al., 2010) and does not conform to the logical progression of topics necessary for appropriate acquisition of core content. In this respect, the treatment of rational number concepts in the Lebanese traditional mathematics curriculum is not consistent with

requirements proposed by the CRF (Clements and Sarama, 2007). For example, there is a heavy emphasis on mastering the procedures of operations on fractions before a strong conceptual understanding of a 'unit' is grounded. The mathematical activities offered, are mostly one-step paper-and-pencil exercises that focus on algorithmic fluency directed mainly toward formalization of mathematics rather than mathematical inquiry.

Furthermore, the lack of updated technological and pedagogical materials and resources that facilitate hands-on activities does not adequately support the acquisition of concepts and skills. In terms of classroom practice, and due to relatively large classroom sizes, individual work is highly encouraged and response to individual differences is relatively weak (World Bank Group, 2006). A synopsis of the basic differences between RNP and TRAD curricula is depicted in Table 1.

Table 1. Basic differences between RNP and TRAD curricula.

RNP curriculum (grade 5)	TRAD curriculum (grade 5)
• Capitalizes on the use of situated modalities: Written inscriptions and symbols; oral speech; physical manipulatives; pictorial displays; and real world contexts	• Employs heavy use of symbolic language; is writing intensive and emphasizes numeric and procedural approaches; physical manipulatives are absent
• Appropriates the sequencing of fraction related concepts based on students' learning trajectories; early emphasis on the development of fraction number sense and concepts of unit and order and equivalence ideas	• Follows a fragmented approach to introducing fraction concepts; naming of fractions and ideas of order and equivalence are briefly presented • Most instructional time allotted to operations on fractions
• Delays the inclusion of operations on fractions to allow time for the ideas of order and equivalence to be grounded	• Prioritizes the teaching of operations on fractions early, addition and subtraction of like and unlike denominators are taught in grade 4 and repeated in grade 5
• Emphasizes the conceptual understanding of fraction operations based on ideas of equivalence through the use of multiple representations and estimation	• Focuses heavily on algorithmic fluency and execution of computations with fractions • Is directed towards formalization of techniques to add/subtract fractions rather than mathematical inquiry
• Limits the size of denominators to 12 or below	• Increases the size of denominators to more than 30

METHOD

This study examines the null hypothesis that 5th grade students taught using the RNP curriculum will not have higher average scores on fraction-related problems than 5th grade students using the TRAD curriculum.

Sample

A sample of five schools was randomly selected from 14 K-12 schools. The selected schools are labeled A through E for reference. Each teacher at schools A and D taught two grade 5 classes; each teacher at schools B and C taught one grade 5 class; and the single teacher at school E taught all four grade 5 classes. Half of the classrooms at each school were randomly assigned to receive the RNP curriculum and the other half were designated as control, i.e., TRAD classrooms, with the exception of school E. In school E, two of the four classes were randomly selected and assigned as RNP classes and no control was taken from this school as it was not admissible to have a single teacher teach both RNP and TRAD classes. The distribution of grade five classrooms, teachers and students in each of the five participating schools between RNP and TRAD groups is given in Table 2.

Intact classrooms were used and were randomly assigned as RNP or TRAD based on the random assignment of teachers into a particular group. Hence, students taught by RNP teachers (i.e., teachers who implemented the RNP curriculum) were designated as RNP students, and those taught by TRAD teachers as TRAD students. Based on the official records of the schools central administration, at the beginning of the academic year, students within a grade are randomly placed in prospective sections irrespective of their achievement level. Every year, teachers are expected to teach a different class within each cycle (i.e., elementary, middle, and secondary). The distribution of grade five classrooms, teachers, and students in each of the five participating schools between RNP and TRAD groups is given in Table 2.

Table 2. Distribution of grade five classrooms, teachers, and students in five schools between RNP and TRAD groups.

	RNP Classrooms	Teachers	Students	TRAD Classrooms	Teachers	Students
School A	1	1	25	1	1	26
School B	2	1	52	2	1	46
School C	2	1	52	2	1	37
School D	3	3	87	3	3	114
School E	2	1	69	–	–	–
Total	10	7	285	8	6	223

Teacher training

Training sessions were held independently for RNP teachers (i.e., teachers using the RNP curriculum, and the TRAD teachers (i.e., those using the Lebanese traditional curriculum). During the course of training, teachers became more familiar with the RNP curriculum as they experienced the material as learners, practicing various teaching strategies proposed by the RNP teachers' lesson plan guide which they were to use with their students (Chahine, in press). RNP teachers were given RNP lesson plans and materials at the end of the first two-hour session of orientation. During, and before the end of the study, teachers participated in two additional two-hour training sessions to reinforce and support their delivery of RNP lesson plans during the period of the study.

Training for teachers in the control group comprised study group discussions and reflections on episodes of teaching problem situations that involve fraction concepts.

Data collection

Two instruments were used to collect quantitative data: the RNP test and a school-generated assessment (TRAD test) that covered similar content as the RNP test. Both the RNP and TRAD groups were tested using the RNP test and the TRAD test. Both tests were written in Arabic, the language used in the study.

Instruments

RNP test

The test consisted of 31 items, which included word problems that required students' written explanations as well as pictorial drawings that elicited short answers (see Fig. 1).

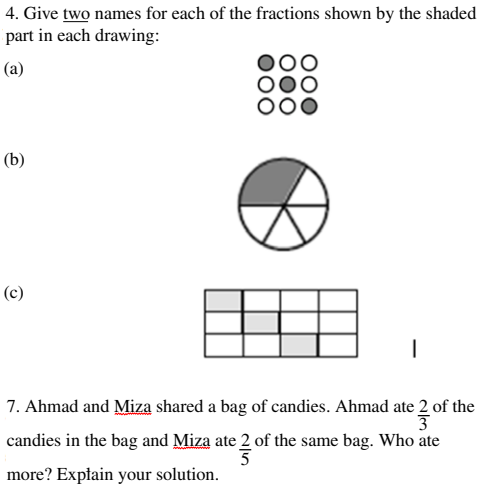

Figure. 1 Two test items included on the RNP test.

TRAD test

The test included 17 items, most of which required one answer and covered the same content as the RNP test. Two sample test items included on the school test are shown in Fig. 2.

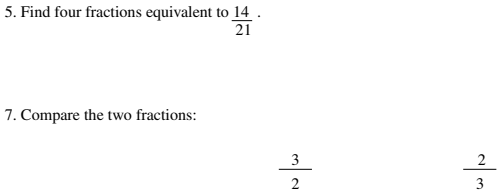

Figure. 2 Two sample test items included on the TRAD test.

RESULTS

Three analyses were conducted on students' quantitative data: reliability, fidelity and MANOVA.

Reliability and content validity of RNP and TRAD tests

The content validity of RNP test was assumed based on experts' judgments. One of the co-authors, a key researcher and co-principal investigator (1979 — present) of the Rational Number Project, supervised the implementation of the study. In a similar vein, it was also assumed that, having been written by subject matter coordinators, the TRAD test passed the content validity measure. Cronbach's alpha coefficient was used to measure the reliability of both the RNP test and the TRAD test. A significantly high Cronbach's alpha of 0.85 was found for the 31 items on RNP test and a value of 0.81 for the 17 items of the school test (Chahine, in press). Point-biserial correlations for all items on the RNP test show acceptable positive point-biserial values (i.e., greater than 0.05), which indicates that the students with high scores on the overall test tended to answer the items correctly and that students with low scores tended to answer the items incorrectly. A similar result was found for the TRAD test.

Fidelity analysis

Implementing the RNP curriculum with fidelity was an essential component in evaluating the effectiveness of curricular intervention as reflected in students' performance. Unscheduled school visits and videotaped classroom observations helped capture not only classroom enactments of the RNP curriculum, but also provided evidence on the dynamics of students' interactions with the assigned activities and their responsiveness to the materials.

To ensure fidelity, measures were proposed related to the method of implementation, frequency, and support systems (NRCLD, 2006). Direct observations and informal conversations with teachers and students, as well as evidence from students' work samples, affirmed the quality of implementation. An average of one classroom observation in four lessons was conducted during the six-week period of the study. Additionally, ensuring the availability of resources for the RNP lessons, as well as providing ongoing professional training for teachers, supported the intended quality and level of implementation of the curriculum.

MANOVA

Results of the RNP and TRAD tests were analyzed using a mixed between-within-subjects MANOVA to test for significant differences in achievement between students using the RNP curriculum and those using the TRAD curriculum. The between-subjects independent variable represented the type of curriculum (RNP or TRAD) and a within-subjects independent variable the type of test (RNP or TRAD). The goal for using a mixed between-within-subjects design was to determine whether the performance of grade five students, as reflected by their scores on the RNP test and the TRAD test, was affected by the type of curriculum used.

The MANOVA conducted to test the proposed hypothesis produced a statistically significant main effect for the type of test (Wilk's Lambda $= 0.98$, $F\,(1, 501) = 11.35$, $p = 0.001$). The multivariate $\eta^2 = 0.02$ showed a small effect size and indicated that approximately 2% of the multivariate variance in test scores was associated with the difference in scores on the TRAD test and the RNP test. Overall, students' average percent correct score was higher on the RNP test (71.62%) than on the school test (68.16%).

Additionally, the interaction effect between curriculum and type of test was also statistically significant (Wilk's Lambda $= 0.99$, $F\,(1, 501) = 5.17$, $p = 0.023$) with a multivariate $\eta^2 = 0.01$ indicating a small effect size. This suggested that about 1% of the multivariate variance in test scores

was associated with the interaction between group and test. As the study had a relatively large sample size ($N = 508$), it can be argued, that a very small effect size as given by $\eta^2 = 0.01$ can be considered statistically significant in a practical sense. In the context of this study, the small effect size can be attributed to the relatively short duration of the study; six weeks may not be long enough to substantiate change in performance (Hill et al., 2008).

To explore the nature of this interaction (see Fig. 3), simple effects analyses were run to examine the mean differences among test scores on RNP and TRAD tests. A separate one-way MANOVA, with type of test as a within-subjects factor, was run for each curriculum. After splitting the sample by group, the mean difference between scores on the RNP and TRAD tests was determined first for the TRAD group and then for the RNP group.

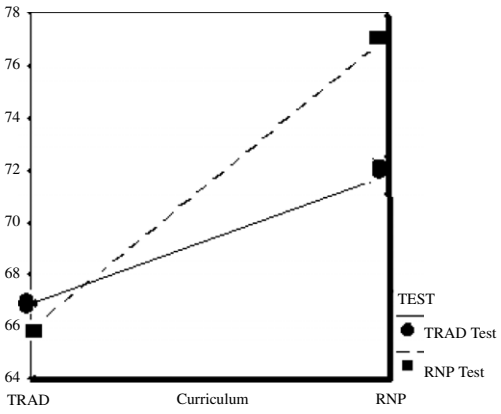

Figure. 3 SPSS output of a profile plot indicating the effect of interaction between Group and Test type.

Results of the analysis of the simple effects produced a statistically significant difference between mean scores on the RNP and TRAD tests for the RNP group ($p < 0.001$), but not for the TRAD group ($p = 0.378$). While students who took the RNP curriculum scored higher on the RNP test than the TRAD test, the TRAD students had similar mean scores on both tests (see Table 3).

Table 3. Means and standard deviations of RNP and TRAD scores on RNP test and school test.

Test	Group	Mean	Standard deviation
RNP	RNP	76.23	15.59
	TRAD	65.75	16.67
TRAD	RNP	70.86	23.54
	TRAD	64.71	24.90

To explore further the nature of the interaction, a second simple effects analysis was conducted that tested the difference between the RNP and the TRAD groups on each test. For both the TRAD test ($p = 0.005$) and the RNP test ($p < 0.001$), the difference was found to be statistically significant, with the RNP group scoring higher, on average, than the TRAD group. Therefore, the RNP group not only did better on the RNP test but also on the TRAD test compared to the TRAD group.

DISCUSSION

The findings of this research support previous positions in mathematics education literature that encourage the development of conceptual understanding prior to mastering computational fluency (Bezuk and Cramer, 1989; Cramer et al., 2002; NCTM, 1991). As shown in this study, overreliance on formalization of fraction concepts before building a conceptual understanding of their underlying properties often hindered successful problem solving. When encountering a question that is beyond their current understanding, traditionally taught students tended to resort to procedures devoid of understanding to find an answer to the question. This, in fact, mirrors earlier findings that investigated the performance of US students in RNP research (Bezuk and Cramer, 1989). On the other hand, exposing RNP students to a well designed instruction using multiple representations did, to a large extent, mediate the way they constructed their understanding of the basic fraction related concepts.

It has been argued, in other relevant studies, that a focus on 'unit' is crucial in linking students understanding of whole numbers and fractions (Behr et al., 1994; Lee and Szatin, 2008; Petit et al., 2010). In the Lebanese traditional curriculum, the concept of unit is neither explicitly dealt with, nor included as an independent component in the textbooks and consequently, not taught in a systematic manner. As was evident in this study, TRAD students who failed to compare fractions successfully had not yet internalized and consolidated the concept of unit. They did not discern whether two fractions to be compared were derived from the same unit and by inference did not realize that fractions have meaning only in relation to the whole from which they are derived.

The paper has attempted to highlight the importance of learning fractions with understanding as a facilitator to problem solving. It has been shown in this study that engaging students in a well-designed instruction that incorporates various representational tools and resources, positively impacted the way they learned and internalized rational number concepts. It is therefore contended, that unless one allows them the opportunity to foster meaningful conceptualization prior to symbolic manipulation, students' problem solving and reasoning skills will be highly jeopardized.

Limitations of the study

It is appropriate to acknowledge a number of delimitations and limitations that may have affected the current study. Firstly, generalization beyond the population of selected schools is an issue to be considered. Secondly, the necessity of using intact classes instead of a random assignment of students into interventions might have affected the internal validity of the study design. Thirdly, the fidelity of implementing the RNP lessons equally across the 10 treatment groups might be another issue to consider. McMillan (2007) notes that "it is simply not realistic to assume that interventions are standardized, even if there is a detailed protocol and experimenters do not make mistakes in the intervention" (p. 3).

Finally, limitations may have arisen due to instrumentation. On one hand, there is a possibility of a measurement bias because the study employed two tests, RNP and TRAD. Students' responses might have been influenced by the variety in the two tests employed. Additionally, the RNP test included test items that were extracted from previous RNP research and appropriated to the Lebanese cultural and educational context. Care was taken to keep RNP materials as intact as possible during the process of translation and customization however, such arrangements might not absolve the constraints posed by modifications in response to certain language considerations.

FUTURE PROSPECTS

Further longitudinal research on teaching and learning fraction related concepts in the Lebanese classroom is needed. The gist of the proposed research was to identify linkages between curricular reform and best practices that foster students' development of high-order thinking processes that are sensitized to the Lebanese context.

A broad range of research has highlighted the importance of culturally relevant teaching in enhancing students' learning. The findings of this study are consistent with recommendations supporting the use of research-based instructional practices that foster a robust curriculum, accommodating the needs of all students. Boaler and Staples (2008) assert that: "Students' opportunities to learn are significantly shaped by the curriculum used in the classrooms and by the decisions teachers make as they enact curriculum and organize other aspects of instruction" (p. 610). Furthermore, research studying the effects of reform curricula, which are conceptually focused and well-structured, indicated higher and more equitable results for all the students (Boaler, 2002; Post et al., 2008; Schoenfeld, 2002).

Extensive international research has concluded that fraction concepts are complex and that students generally have difficulty in shifting their understanding from whole numbers to fractions (Charalambous and Pitta-Pantazi, 2007). Notwithstanding the efforts that have been made to reform fraction instruction, rote teaching and learning remains a major cause of low performance. In addition to the complex nature of fraction concepts, the most commonly cited explanation for students' failure in understanding fractions are an overreliance on symbol manipulation and routinized procedures. In this study, a perspective on students' performance with fraction concepts from an international perspective was provided. In Lebanon, rote-acquisition approaches to instruction are traditionally the dominant mechanism for teaching mathematics. Large-scale future challenges will be to help teachers to sublimate their role as knowledge dispensers and adopt a more experimental and cognitive-based perspective.

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