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Effect of thermal conductivity on gravitational instability of quantum plasma having fine dust particles

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ABSTRACT

Effect of thermal conductivity on gravitational instability of quantum plasma in the presence of fine dust particles has been investigated. Following the linearized stability theory and normal mode analysis, the paper established a general dispersion relation of the problem. Modified condition of Jeans gravitational instability is obtained due to quantum effect. Numerical calculations were performed to find the effect of each parameter on the growth rate of instability. The effect of fine dust particles does not affect the instability condition of the system but stabilizes the system by decreasing the growth rate of unstable mode. Curves show the destabilizing effect of thermal conductivity and stabilizing effect of quantum correction on the growth rate of unstable mode. The stability of the system is discussed by Routh-Hurwitz criterion of stability.

Keywords: gravitational instability, fine-dust particles, thermal conductivity, quantum correction

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1. INTRODUCTION

In recent years, numerous researchers have been carrying out investigations on various salient features of self-gravitational instability of the gaseous plasma contaminated by the different parameters encountered very often in space and laboratory plasmas. The gravitational instability of ideal plasma has been discussed by Chandrashekhar.[1] He has considered hydromagnetic stability of self-gravitating, unbounded, homogeneous, rotating plasma of infinite conductivity. His results show that the presence of steady magnetic field and uniform rotation does not affect the Jeans stability criterion. Yang et al.[2] have investigated the large–scale gravitational instability and star formation in molecular clouds. Prajapati et al.[3] have investigated the problem of self-gravitational instability of rotating viscous Hall plasma arbitrary radiative heat-loss functions and electron inertia. Ali and Shukla[4] have studied the Jeans instability in plasma with positive negative charged and neutral dust components. Jacobs and Shukla[5] have investigated the problem of self-gravitational instability of partially ionized astrophysical plasma embedded in a large-scale magnetic field. Salimullah et al.[6] have studied the Jeans instability of quantum dusty magnetoplasma. Thus we find that the problem of self-gravitational instability of astrophysical gaseous and dusty plasmas is discussed in various ways by many investigators taking different assumptions and parameters.

The importance of thermal conductivity, as it is associated with most of the astrophysical situations, is an established fact. The problem of thermal instability, arising owing to various heat mechanism of interstellar matter, plays an important role in astrophysical condensations and the formation of prominences through condensation of coronal material. Coroniti[7] investigated the dissipative effects of viscosity, finite electrical and thermal conductivities on shock waves. Kato and Kumar[8] have studied the problem of the gas plasma incorporating finite thermal conductivity; and in their results, they found that adiabatic speed of sound is being replaced by the isothermal one, much similar to what happens in the absence of magnetic field. Chhajlani and Vyas[9] have investigated the effects of thermal conductivity and suspended particles on the gravitational instability of magnetized rotating plasma through porous medium. Recently, Shaikh and Khan[10] have discussed the instability of thermally conducting self-gravitating system. Thus, due to the importance of thermal conductivity in the study of self-gravitational instability of gaseous plasma we are willing to take its consideration in the present study along with quantum effects and fine dust particles.

In recent interstellar medium (ISM) observations, it has been established that comets consist of a dusty ‘Snowball’ of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice versa. Sharma and Sharma et al.[11–14] have investigated the effect of fine dust particles (suspended particles) on the onset of Benard convection in hydromagnetics incorporating various parameters. The importance of suspended particles in the study of gravitational instability of magnetized and rotating plasma has been studied by Chhajlani and Sanghvi.[15] Pensia et al.[16] have investigated the role of Coriolis force and suspended particles in the fragmentation of mater in the central region of galaxy and suggested that Coriolis force and suspended particles play important role in the central region of galaxy. The effect of porosity and suspended particles on Jeans instability under thermal effect has been studied by Pensia et al.[17] Thus the aim of the present paper is to study the effect of thermal conductivity on self-gravitational instability of an infinite homogeneous magnetized quantum plasma in the presence of fine dust particles.

The problem of thermal and self-gravitational instability in quantum plasmas has been an important area of research in the recent years. The study of quantum effects in plasma becomes important when the de Broglie of the charge carriers is equal or greater than the interparticle distance. The quantum correction in gaseous plasma was first studied by Pines.[18,19] Gardner[20] has given the quantum hydro-dynamic (QHD) model for semi-conductor physics to describe the transport of charge, momentum and energy in plasmas. The quantum magneto-hydrodynamic (QMHD) model was obtained by Haas[21] with the help of QHD model with magnetic field based on the Wigner–Maxwell equations. Lundin et al.[22] used the QMHD model to investigate the problem of Jeans instability of spin quantum plasma in the presence of a magnetic field. Ren et al.[23] studied the effect of electrical resistivity on Jeans instability of quantum magnetoplasma. Shukla and Stenflo[24] investigated the Jeans instability of self-gravitating astrophysical quantum dusty plasma. The effect of Hall current on Jeans instability viscous quantum plasma in the presence of a magnetic field was examined by Prajapati and Chhajlani.[25] Thus in the present analysis we will apply the QMHD model on self gravitating, and thermally conducting plasma having fine dust particles.

It is clear, from all the above studies, that none of the authors have carried the joint study of the effects of fine dust particles, thermal conductivity and quantum correction on the problem of self-gravitational instability of gaseous plasma. Thus, in the present work, we are motivated to investigate the effect of thermal conductivity on self-gravitational instability of gaseous quantum plasma in the presence of fine dust particles. The result of the present study will help to understand the interstellar medium structure.

2. LINEARIZED PERTURBATION EQUATIONS

We consider an infinite homogeneous viscous, self-gravitating quantum plasma including thermal conductivity, and fine dust particles.

If we assume uniform particle size, spherical shape and small relative velocities between the two phases, then the net effect of particles in the gas is equivalent to an extra body force term per unit volume $K_s N(\vec{v} - \vec{u})$ and is added to the momentum transfer equation for gas, where K_s the constant given by Stokes' drag formula $K_s = 6\pi\rho\nu r$, r being the particle radius, ν is the kinetic viscosity of clean gas, ρ and N represents the density of gas and the number density of particles, respectively. $\vec{u}$, and $\vec{v}$, denotes the gas and particle velocity, respectively. Self-gravitational attraction U is added along with the kinetic viscosity term in the equation of motion for gas. In writing the equation of motion, for the particles, we neglect the buoyancy force as its stabilizing effect, for the case of two free boundaries is extremely small. Inter particle reactions are also ignored by assuming the distance between particles to be too large compared with their diameters.

The QMHD model is considered as given by Haas[21] with thermal conductivity and fine dust particles,[9] let the perturbation of the type $q = q + \delta q$, and perturbations in fluid pressure, fluid density, fluid velocity, temperature and gravitational potential is given by δp, $\delta\rho$, $\delta\vec{u}(u_x, u_y, u_z)$, δT and δU, respectively. Thus we construct the following set of linearized equations consisting momentum transfer for fluid and particles, continuity, thermal energy equation, Poisson equation, and gas equation:

$$\rho\frac{\partial\delta\vec{u}}{\partial t} = -\vec{\nabla}\delta p + \vec{\nabla}\delta U + K_s N(\vec{v} - \vec{u}) + \rho\nu(\nabla^2\vec{v}) + \frac{\hbar^2}{4m_e m_i}\vec{\nabla}(\nabla^2\delta\rho). \tag{1}$$

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\vec{v} = \vec{u}. \tag{2}$$

$$\frac{\partial\rho}{\partial t} + \rho\vec{\nabla}.\vec{u} = 0. \tag{3}$$

$$\rho C_p\frac{\partial}{\partial \mathrm{t}}\delta T - \frac{\partial}{\partial t}\delta p = \lambda\nabla^2\delta T. \tag{4}$$

$$\nabla^2\delta U = -4\pi G\delta\rho. \tag{5}$$

$$\frac{\delta T}{T} + \frac{\delta\rho}{\rho} = \frac{\delta p}{p}. \tag{6}$$

where $\tau = m/K_s$, and the parameters G, p, T, C_p, λ, R, $\hbar = h/2\pi$, respectively denotes the gravitational constant, pressure, temperature, specific heat at constant pressure, coefficient of thermal conductivity, gas constant, and the Planck constant divided by 2π.

3. DISPERSION RELATION

Let us consider plane waves propagated in the X and Z-directions, so that all perturbed quantities vary as

$$\exp\{i(k_x x + k_z z + \sigma t)\}. \tag{7}$$

where σ is the frequency of harmonic disturbances, $k_{x,z,}$ are wave numbers in X and Z direction, respectively, such that $k_x^2 + k_z^2 = k^2$. For perturbation of the form (7), using (2) to (6) the algebraic amplitude of equation (1) can be written as

$$\left[-\tau\sigma^2 + i\sigma\alpha + \nu k^2\right]u_x + \frac{ik_x}{k^2}\beta\left(\frac{\sigma\Omega_j^2 + \Omega_k\Omega_l^2}{\sigma + \Omega_k}\right)s = 0 \tag{8}$$

$$\left[-\tau\sigma^2 + i\sigma\alpha + \nu k^2\right]u_y = 0. \tag{9}$$

$$\left[-\tau\sigma^2 + i\sigma\alpha + \nu k^2\right]u_z + \frac{ik_z}{k^2}\beta\left(\frac{\sigma\Omega_j^2 + \Omega_k\Omega_l^2}{\sigma + \Omega_k}\right)s = 0. \tag{10}$$

Now taking the divergence of equation (1) using (2) to (6) we get as

$$\left[-i\tau\sigma^3 - \sigma^2\alpha + i\sigma\left\{\left(\frac{\sigma\Omega_j^2 + \Omega_k\Omega_l^2}{\sigma + \Omega_k}\right)\tau + \nu k^2\right\} + \left(\frac{\sigma\Omega_j^2 + \Omega_k\Omega_l^2}{\sigma + \Omega_k}\right)\right]s = 0 \tag{11}$$

Equations (8)–(11) can be written in the matrix form as

$$[A][B] = 0 \tag{12}$$

where $[A]$ is a single column matrix with elements (u_x, u_y, u_z, s), and $[B]$ is forth order square matrix whose elements are

$$X_{11} = \left[-\tau\sigma^2 + i\sigma\alpha + \nu k^2\right]. \quad X_{12} = 0. \quad X_{13} = 0. \quad X_{14} = \frac{ik_x}{k^2}\left(\frac{\sigma\Omega_j^2 + \Omega_k\Omega_l^2}{\sigma + \Omega_k}\right)\beta. \quad X_{21} = 0.$$

$$X_{22} = \left[-\tau\sigma^2 + i\sigma\alpha + \nu k^2\right]. \quad X_{23} = 0. \quad X_{24} = 0. \quad X_{31} = 0. \quad X_{32} = 0.$$

$$X_{33} = \left[-\tau\sigma^2 + i\sigma\alpha + \nu k^2\right]. \quad X_{34} = \frac{ik_z}{k^2}\left(\frac{\sigma\Omega_j^2 + \Omega_k\Omega_l^2}{\sigma + \Omega_k}\right)\beta. \quad X_{41} = 0. \quad X_{42} = 0. \quad X_{43} = 0.$$

$$X_{44} = \left[-i\tau\sigma^3 - \sigma^2\alpha + i\sigma\left\{\left(\frac{\sigma\Omega_j^2 + \Omega_k\Omega_l^2}{\sigma + \Omega_k}\right)\tau + \nu k^2\right\} + \left(\frac{\sigma\Omega_j^2 + \Omega_k\Omega_l^2}{\sigma + \Omega_k}\right)\right].$$

$$\alpha = \left(1 + \nu k^2\tau + \frac{K_s N\tau}{\rho}\right),$$

$$\beta = (1 + \sigma\tau), \quad \Omega_l^2 = \left(k^2c'2 - 4\pi G\rho + \frac{\hbar^2k^4}{4m_em_i}\right), \quad \Omega_J^2 = \left(\Omega_j^2 + \frac{\hbar^2k^4}{4m_em_i}\right).$$

$$\Omega_j^2 = \left(k^2c^2 - 4\pi G\rho\right)$$

$\Omega_k = \left(\lambda\gamma k^2/\rho c_p\right)$, where $c^2 = \gamma c'2$, is the adiabatic velocity of sound, $c' = \sqrt{p/\rho}$, is the isothermal velocity of the sound, c_p is the specific heat at constant pressure $s = \delta\rho/\rho$ is the condensation of the medium.

For a nontrivial solution of equation (12) the determinant of the square matrix on the left hand side should vanish, leading to the dispersion relation.

$$\left[-\tau\sigma^2 + i\sigma\alpha + \nu k^2\right]^3 \times \left[-i\tau\sigma^3 - \sigma^2\alpha + i\sigma\left\{\left(\frac{\sigma\Omega_j^2 + \Omega_k\Omega_l^2}{\sigma + \Omega_k}\right)\tau + \nu k^2\right\} + \left(\frac{\sigma\Omega_j^2 + \Omega_k\Omega_l^2}{\sigma + \Omega_k}\right)\right] = 0. \tag{13}$$

Equation (13) represents the desired dispersion relation for an infinitely extending, self-gravitating viscous plasma having fine dust particles under the influence of thermal conductivity and quantum corrections. We find that, in this dispersion relation, the term due to the thermal conductivity have entered through the factor Ω_k and that the term due to the quantum correction have entered through the factor $\left(\hbar^2k^4/4m_em_i\right)$. If we ignore the effects of quantum correction and thermal conductivity then (13) reduces to Sharma[11] and also reduces to Chhajlani and Sanghavi[15] obtained for non-rotating unmagnetized plasma. Again in the absence of thermal conductivity, viscosity and fine dust particles the preceding dispersion relation reduces to Ren et al.[23] on ignoring the effects of magnetic field and electrical resistivity in their case.

Thus with these correlations we find that the dispersion relation (13) is modified due to the effects of quantum correction, thermal conductivity and fine dust particles. The above dispersion relation (13) can alternatively be subjected to the following two conditions with substitutions for Ω_j^2, Ω_I^2, α, and $i\sigma = \omega$.

$$\tau\omega^2 + \omega\left(1 + \nu k^2\tau + \frac{K_S N\tau}{\rho}\right) + \nu k^2 = 0. \tag{14}$$

This dispersion relation shows the combined influence of kinematic viscosity and fine dust particles on waves propagating in hydro-magnetic fluid plasma. This dispersion relation is similar to that of already obtained by Sharma[11] and Chhajlani and Vyas.[9] Since all the coefficients and the constant term of (14) are positive hence, following Descartes' rule, it does not have a positive real root or a complex root whose real part is positive, and so the system is stable. Thus, we conclude that the viscous force is capable to stabilize the system while the presence of fine dust particles increases this effect.
The second factor of equation (13) equating to zero, we obtain as

$$\begin{aligned}&\tau\omega^4 + \omega^3\left[1 + \tau\left(\nu k^2 + \frac{K_S N}{\rho} + \Omega_k\right)\right] + \omega^2\left[\Omega_k\left(1 + \nu k^2\tau + \frac{K_S N\tau}{\rho}\right) + \Omega_j^2\tau + \nu k^2\right]\\ &+ \omega\left[\Omega_k\tau\Omega_I^2 + \nu k^2\Omega_k + \Omega_j^2\right] + \Omega_k\Omega_I^2 = 0\end{aligned} \tag{15}$$

This represents the dispersion relation for self-gravitating plasma incorporating the effects of kinematic viscosity, thermal conductivity, fine dust particles, and quantum correction. Thus, this is a gravitating mode affected by viscosity, thermal conductivity, the presence of fine dust particles and quantum correction. In the absence of quantum correction, this dispersion relation is similar to that already obtained by Chhajlani and Vyas.[9] For a thermally non-conducting medium without having fine dust particles, the above dispersion relation is reduced to Ren et al.,[23] and Prajapati and Chhajlani[25] for longitudinal direction of propagation. This fourth degree equation (15) may be reduced to particular cases so that the effect of each parameter is discussed separately.

Case I: $- \lambda = Q = 0$. To study the particular cases we first consider thermally non-conducting, gaseous plasma without quantum effect, the dispersion relation (15) reduces to

$$\tau\omega^3 + \omega^2\left(1 + \nu k^2\tau + \frac{K_S N\tau}{\rho}\right) + \omega\left(\Omega_j^2\tau + \nu k^2\right) + \Omega_j^2 = 0 \tag{16}$$

This cubic equation represents a self-gravitating viscous medium having dust particles, and it is identical to equation (16) of Pensia et al.[17] The effect of fine dust particles enters through two parameters, τ (relaxation time) and $(K_S N\tau)/\rho$ which is mass concentration of particles. It is clear from equation (16) that when $\Omega_j^2 < 0$, the product of the roots or at least one root of σ is positive. Hence, the system is unstable. Thus for the cases of equation (16) the condition of instability is

$$\left.\begin{aligned}\Omega_j^2 &= \left(c^2k^2 - 4\pi G\rho\right) < 0\\ k < k_j &= \left(\tfrac{4\pi G\rho}{c^2}\right)^{1/2}\\ \lambda_j &= c\left(\tfrac{\pi}{G\rho}\right)^{1/2}\end{aligned}\right\} \tag{17}$$

where k_j is the Jeans wave number and λ_j is the Jeans length. Equation (17) is an original Jeans expression for gravitational instability. The system is unstable for all Jeans length $\lambda > \lambda_j$, of Jeans wave number $k < k_j$. It is evident from equation (17) that Jeans criterion of instability remains unchanged in the presence of dust particles for viscous medium.

In the ISM, the gravitational unstable modes are responsible for structure formation; thus, the choice of the arbitrary values of relaxation time τ and Stokes' drag K_S parameters in the present problem in order to study the effect fine dust particles on the growth rate of an unstable mode. To solve the dispersion relation (16) numerically we introduce the dimensionless parameters (Appendix A) in terms of self-gravitation.

In Figures 1 and 2, we have depicted the dimensionless growth rate versus dimensionless wave number, for various arbitrary values of the Stokes' drag K_s^* constant and τ.

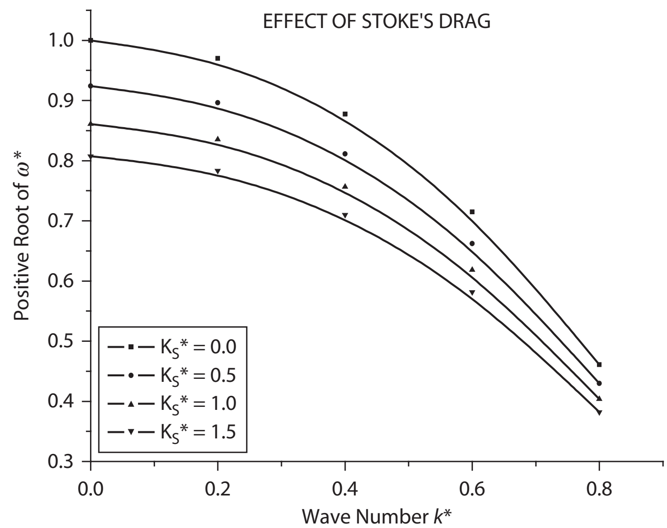

Figure 1. The growth rate of instability is plotted against the dimensionless wave number k^* with variation in the Stokes' drag constant $K_s^* = 0.0, 0.5, 1.0, 1.5$, with taking the values of ν^* and τ as unity.

Figure 1 is plotted for the growth rate of instability (positive imaginary root of ω^*) against the dimensionless wave number k^* with variation in the Stokes' drag constant $K_s^* = 0.0, 0.5, 1.0, 1.5$, with taking the values of ν^* and τ as unity.

From the curves we find that, due to an increase in the Stokes' drag constant parameter, the growth rate of instability decreases. The peak value of the growth rate is affected by the presence of the Stokes' drag constant parameter and it is different for different values of K_s^*. However, it may be noted that Stokes' drag constant parameter tends to stabilize the configuration.

In Figure 2, we have plotted for the growth rate of an unstable mode (positive imaginary root of ω^*) against the dimensionless wave number k^* with variation in the value of $\tau = 0.5, 1.0, 1.5, 2.0$, with taking the values of ν^* and K_s^* as unity.

From Figure 2, it is observed that the τ has a similar effect on the growth rate compared to that of the Stokes' drag constant K_s^*. In other word, due to an increse in the value of τ, the growth rate of the

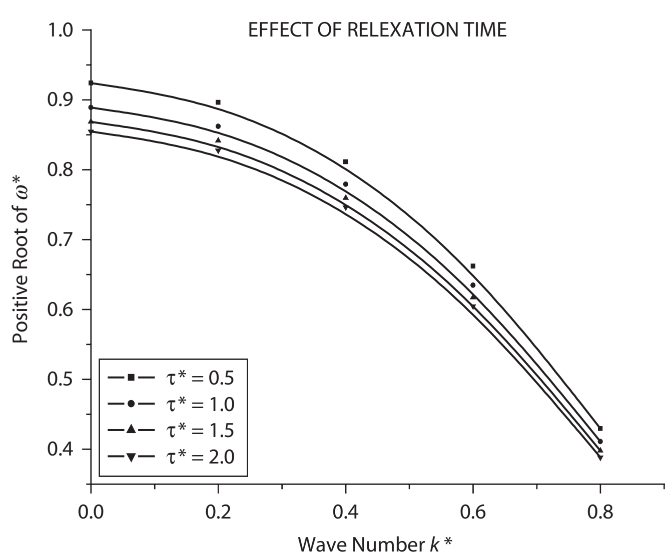

Figure 2. The growth rate of instability is plotted against the dimensionless wave number k^* with variation in the value of $\tau = 0.5, 1.0, 1.5$, and 2.0, with taking the values of ν^* and K_s^* as unity.

instability decreases. Hence the value of τ has a stabilizing influence on the growth rate of the instability.

Case II: $-\ \lambda = 0,\ Q \neq 0$. In the case of thermally non-conducting, self-gravitating quantum plasma having fine dust particles, the dispersion relation (15) reduces to

$$\tau\omega^3 + \omega^2\left(1 + \nu k^2\tau + \frac{K_s N\tau}{\rho}\right) + \omega\left(\Omega_j^2\tau + \nu k^2\right) + \Omega_j^2 = 0. \tag{18}$$

This dispersion relation (18) shows the combined influence of viscosity, quantum correction and the effect of the fine dust particles on self-gravitational instability of gaseous plasma. The modified Jeans instability criterion in terms of quantum correction can be easily obtained from the constant term of equation (18) and is given by

$$\left[k^2c^2 + \frac{\hbar^2k^4}{4m_e m_i} - 4\pi G\rho\right] < 0. \tag{19}$$

The expression of critical Jeans wave number and the corresponding Jeans length is given by

$$k_{j1} = k_j\left[1 + \frac{\hbar^2k^2}{4m_e m_i c^2}\right]^{-1/2}, \quad \lambda_{j1} = \lambda_j\left[1 + \frac{\hbar^2\pi^2}{m_e m_i c^2\lambda^2}\right]^{1/2}. \tag{20}$$

Thus the system will be unstable for all wave numbers $k < k_{j1}$, (where k_{j1} is critical Jeans wave number affected by quantum correction) given by equation (20). The above condition of instability (19) is coincident with that found by Ren et al.[23] and Prajapati and Chhajlani.[25] From equation (20), it is obvious that the quantum correction term couples with the adiabatic sound velocity and decreases the Jeans wave number. Thus, the effect of quantum term is to stabilize the system.

The stabilizing effect of quantum correction will be verified by solving the dispersion relation (15) numerically using the dimensionless quantities.

Figure 3 is plotted for the growth rate of instability (positive imaginary root of ω^*) against the dimensionless wave number k^* with variation in the quantum correction $Q^* = 0.0, 0.5, 1.0, 1.5$, with taking the values of ν^*, τ, and K_s^* as unity.

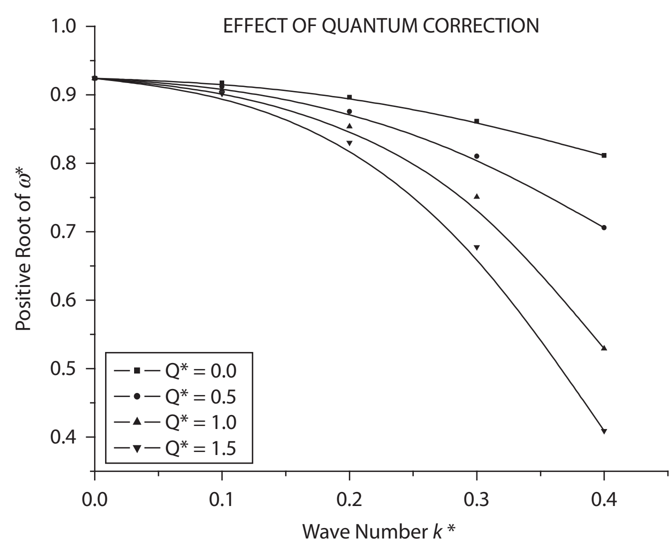

Figure 3. The growth rate of instability is plotted against the dimensionless wave number k^* with variation in the quantum correction $Q^* = 0.0, 0.5, 1.0, 1.5$, with taking the values of ν^*, τ, and K_s^* as unity.

Figure 3 shows the variation in growth rate with respect to quantum correction. Here we notice that when the system is classical ($Q^* = 0$), the growth of instability is maximum while the growth rate decreases with the increasing value of quantum correction ($Q^* = 0.5, 1.0, 1.5$). Thus, from Figure 3, we conclude that the effect of quantum is to stabilize the system.

Case III: $-\ \lambda = Q \neq 0$. For this case of viscous self-gravitating quantum plasma subject to thermal conduction in the presence of fine dust particles, the original dispersion relation represented by the

(15) remains unchanged, and the condition of instability obtained from constant term of the last coefficient

$$\left(k^2c'2 - 4\pi G\rho + \frac{\hbar^2 k^4}{4m_e m_i}\right) < 0 \tag{21}$$

Critical Jeans wave number and Jeans wavelength is given by

$$k < k_{j2} = \sqrt{\frac{4\pi G\rho}{\left(c'^2 + \frac{\hbar^2 k^2}{4m_e m_i}\right)}}. \quad \lambda_{j2} = \lambda_j \left[\frac{\left(1 + \frac{\hbar^2 \pi^2}{m_e m_i c^2 \lambda^2}\right)}{\gamma}\right]^{1/2} \tag{22}$$

This is the new condition of gravitational instability found in present analysis due to inclusion of thermal conductivity and quantum correction. It may be noted that the condition of gravitational instability does not involve the effect of viscosity and fine dust particles. Thus, the Jeans criterion of instability remains valid but the expression of the critical Jeans wave number is modified by considering quantum correction and thermal conductivity but not affected by the presence of fine dust particles. On comparing (15) and (18), we see that due to the inclusion of thermal conductivity, third ordered equation changes to the fourth ordered equation, means one mode is increased, and the adiabatic sonic speed is replaced by isothermal sonic speed in the expression of critical wave number.

In the present case, we have considered the effects of fine dust particles and thermal conductivity parameters, but Ren et al.[23] and Prajapati and Chhajlani[25] have not considered these parameters. Thus, the dispersion relation in the present analysis is modified due to the simultaneous inclusion of fine dust particles and thermal conductivity, and the condition of gravitational instability is modified by the presence of thermal conductivity. Thus fine dust particles have no role to play in the condition of gravitational instability. From equations (22) and (20) we obtain

$$\lambda_{j2} = \lambda_{j1}\left(\frac{1}{\gamma}\right)^{1/2} \tag{23}$$

It is clear from equation (23) that the Jeans length is reduced due to thermal conduction [as $\gamma > 1$], thus the system is destabilized.

To conform, the destabilizing effect thermal conductivity on the growth rate of instability we solve (15) numerically by introducing the dimensionless quantities (appendix A) in terms of self-gravitation.

Figure 4 is plotted for the growth rate of instability (positive imaginary root of ω^*) against the dimensionless wave number k^* with variation in the thermal conductivity $\lambda^* = 0.0, 0.5, 1.0, 1.5$, with taking the values of ν^*, τ, K_s^* and Q^* as unity.

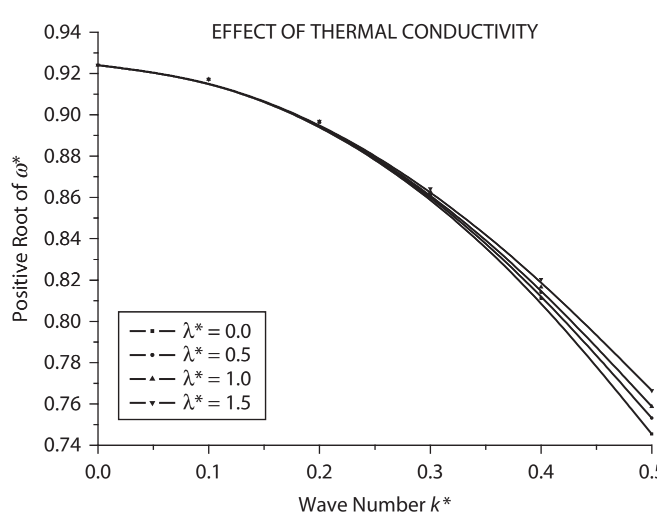

Figure 4. The growth rate of instability is plotted against the dimensionless wave number k^* with variation in the thermal conductivity $\lambda^* = 0.0, 0.5, 1.0, 1.5$, with taking the values of ν^*, τ, K_s^* and Q^* as unity.

Figure 4 shows the effect of thermal conductivity on the growth rate of instability. Here we see that the increasing value of thermal conductivity increases the growth rate of instability. Thus, the thermal conductivity shows a reverse effect, in comparison to quantum correction, on the growth rate of instability and thus destabilizes the system.

In order to discuss the dynamical stability of the system represented by (15), we apply the Routh-Hurwitz criterion. According to this criterion all the coefficients of the polynomial equation (15) should be positive. If $\Omega_j^2 > 0$ and $\Omega_I^2 > 0$, then all the coefficients of (15) are positive and the necessary condition for stability is satisfied. To satisfy the sufficient condition we calculate the minors of the Hurwitz matrix formed by these coefficients, which are

$$\begin{aligned}
\Delta_1 &= \left(\frac{1}{\tau} + \Omega_k + \nu k^2 + \frac{K_S N}{\rho}\right) > 0.\\
\Delta_2 &= \frac{1}{\tau}\Delta_1\left[\Omega_k\left(1 + \nu k^2\tau + \frac{K_S N\tau}{\rho}\right) + \Omega_j^2\tau + \nu k^2\right] > 0.\\
\Delta_3 &= \frac{1}{\tau}\Delta_2\left[\Omega_k\tau\Omega_I^2 + \nu k^2\Omega_k + \Omega_j^2\right] > 0.\\
\Delta_4 &= \frac{\Omega_k\Omega_I^2}{\tau}\Delta_3 > 0
\end{aligned} \tag{24}$$

It is clear from (24) that all Δ's are positive. Therefore, the system represented by (15) is stable if the conditions $\Omega_j^2 > 0$ and $\Omega_I^2 > 0$ are satisfied.

4. CONCLUSIONS

In the present paper, we have analyzed the effect of thermal conductivity on the gravitational instability of quantum plasma in the presence of fine dust particles. The general dispersion relation is obtained, which is modified due to the presence of these parameters. We find that the Jeans criterion of gravitational instability remains valid but the expression of the critical Jeans wave number is modified by parameter of quantum correction. Owing to the inclusion of the thermal conductivity, the adiabatic sound velocity is replaced by the isothermal velocity of sound. The effect of fine dust particles does not affect the instability condition of the system but stabilize the system by decreasing the growth rate of unstable mode. The stability of the system is discussed by using Routh-Hurwitz criterion of stability.

APPENDIX A

Numerical calculations were performed, taking $\gamma = 5/3$, to determine the roots of ω^* from dispersion relation (15), (16) and (18) as a function of dimensionless wave number k^* for different values of the various dimensionless parameters defined as

$$\omega^* = \frac{\omega}{(4\pi G\rho)^{1/2}},\quad k^* = \frac{kc}{(4\pi G\rho)^{1/2}},\quad \nu^* = \frac{\nu(4\pi G\rho)^{1/2}}{c^2},\quad \lambda^* = \frac{\lambda(4\pi G\rho)^{1/2}}{\rho c_p c^2},\quad Q^* = \frac{\hbar^2 k_j^2}{4m_e m_i}$$

$$K_s^* = \frac{NK_s}{\rho(4\pi G\rho)^{1/2}},\quad \Omega_j^{*2} = k^{*2} - 1,\quad \Omega_I^{*2} = k^{*2} - \gamma.$$

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REFERENCES

[1] Chandrashekhar S. *Hydrodynamics and Hydromagnetic Stability*. XIII. Oxford: Clarendon Press; 1961:p.577.
[2] Yang C, Gruendl RA, Chu Y, Mac Low M, Fukui Y. Large-scale gravitational instability and star formation in the large magellanic cloud. *Astrophys J*. 2007;671(1):374–379.
[3] Prajapati RP, Pensia RK, Kaothekar S, Chhajlani RK. Self-gravitational instability of rotating viscous Hall plasma with arbitrary radiative heat-loss functions and electron inertia. *Astrophys Space Sci*. 2010;327(1):139–154.
[4] Ali S, Shukla PK. Jeans instability in a plasma with positive-negative charged and neutral dust components. *Phys Scripta*. 2006;73(4):359–363.

[5] Jacobs G, Shukla PK. Stability of molecular clouds in partially ionized self-gravitating space plasmas. *J Plasma Phys*. 2005;71(4):487–493.
[6] Salimullah M, Jamil M, Shah HA, Murtaza G. Jeans instability in a quantum dusty magnetoplasma. *Phys Plasmas*. 2009;16(1):014502.
[7] Coroniti FV. Dissipation discontinuities in hydromagnetic shock waves. *J Plasma Phys*. 1970;4(02):265.
[8] Kato S, Kumar SS. The gravitational instability of an infinite homogeneous medium having viscosity and thermal conductivity. *Publ Astron Soc Jpn*. 1960;12(3):290–292.
[9] Chhajlani RK, Vyas MK. Effect of thermal conductivity on the gravitational instability of a magnetized rotating plasma through a porous medium in the presence of suspended particles. *Astrophys Space Sci*. 1988;145(2):223–240.
[10] Shaikh S, Khan A. Instability of thermally conducting self-gravitating systems. *J Modern Phys*. 2010;01(01):77–82.
[11] Sharma RC, Sharma KC. Suspended particles and the gravitational instability of a rotating plasma. *Astrophys Space Sci*. 1980;71(2):325–332.
[12] Sharma RC. Magneto-gravitational instability and suspended particles. *Astrophys Space Sci*. 1977;46(2):255–259.
[13] Sharma RC. Thermal hydromagnetic instability of a partially ionized medium. *Physica B + C*. 1976;81(1):199–204.
[14] Sharma RC, Prakash K, Dube SN. Effect of suspended particles on the onset of Benard convection in hydromagnetics. *Acta Physica*. 1976;40(1):3–10.
[15] Chhajlani RK, Sanghvi RK. Suspended particles and the gravitational instability of rotating magnetised medium. *Beiträge aus der Plasmaphysik*. 1985;25(6):623–631.
[16] Pensia R, Patidar AK, Shrivastava V, Kumar V, Prajapat V. The role of Coriolis force and suspended particles in the fragmentation of matter in the central region of galaxy. *J Bangladesh Acad Sci*. 2012;36(1):1.
[17] Pensia RK, Prajapat V, Kumar V, Patidar AK, Shrivastava V. Effect of porosity and suspended particles on Jeans instability under thermal effects. *Advan Stud Theoretic Phys*. 2012;23(6):1121–1136.
[18] Pines D. Classical and quantum plasmas. *J Nucl Energy, Part C: Plasma Phys*. 1961;2(1):5–17.
[19] Pines D. *Elementary Excitations in Solids*. Oxford: Westview Press; 1999.
[20] Gardner CL. The quantum hydrodynamic model for semiconductor devices. *SIAM J Appl Math*. 1994;54(2):409–427.
[21] Haas F. A magnetohydrodynamic model for quantum plasmas. *Phys Plasmas*. 2005;12(6):062117.
[22] Lundin J, Marklund M, Brodin G. Modified Jeans instability criteria for magnetized systems. *Phys Plasmas*. 2008;15(7):072116.
[23] Ren H, Wu Z, Cao J, Chu PK. Jeans instability in quantum magnetoplasma with resistive effects. *Phys Plasmas*. 2009;16(7):072101.
[24] Shukla PK, Stenflo L. Jeans instabilities in quantum dusty plasmas. *Phys Lett A*. 2006;355(4–5):378–380.
[25] Prajapati RP, Chhajlani RK. Effect of Hall current on the Jeans instability of magnetized viscous quantum plasma. *Phys Scripta*. 2010;82(5):055003.